

CENTRE OF MASS

1. **Rigid body:**

A body which does not undergo deformation when it is subjected to an external force is called rigid body

2. **Non Rigid body:**

A body which undergoes deformation when subjected to an external force is called non rigid body.

3. A body can execute 3 types of motions

(1) translatory motion (2) rotatory motion
(3) vibratory motion

Translatory motion:

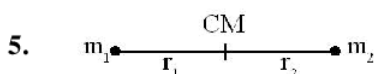
A body in which all the particles same displacement in same time is called translatory motion

Rotatory motion:

A body in which the particles move permanently around a fixed point then the motion of the body is called rotatory motion.

4. **Centre of mass:**

Centre of mass is a point within the boundaries of a system or a body at which its entire mass appears to be concentrated.



When two particles of masses m_1 and m_2 are located on a straight line, their centre of mass lies on the line joining the two particles.

Let r_1 and r_2 be the distances of the particles from their centre of mass respectively, then

$$m_1 r_1 = m_2 r_2$$

6. **Coordinates of centre of mass**

Let us consider a system of n particles of masses $m_1, m_2, \dots, m_n$ whose co-ordinates are $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots, (x_n, y_n, z_n)$ respectively. Then co-ordinates of their centre of mass are

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$x_{cm} = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{m_1 + m_2 + \dots + m_n}$$

$$z_{cm} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

and

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots + m_n z_n}{m_1 + m_2 + \dots + m_n}$$

$$z_{cm} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

7. **Position vector of centre of mass**

Consider two particles of mass m_1 and m_2 whose position are represented by position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively at an instant, then

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

8. **Velocity and acceleration of centre of mass**

Velocity of centre of mass

$$\vec{V}_{cm} = \frac{m_1 \vec{V}_1 + m_2 \vec{V}_2 + \dots + m_n \vec{V}_n}{m_1 + m_2 + \dots + m_n}$$

$$\vec{V}_{cm} = \frac{1}{M} \sum_{i=1}^n m_i \cdot \vec{V}_i$$

9. **Moment of inertia of some bodies of regular shape**

S.No.	Body	Axis	Moment of inertia
1.	Uniform rod of length ℓ	Perpendicular to rod through its center	$\frac{1}{12} M \ell^2$
2.	Uniform rectangular lamina of length ℓ and breadth b	Perpendicular to lamina and through its center	$M \left(\frac{\ell^2 + b^2}{12} \right)$
3.	Uniform circular ring of radius R	Perpendicular to its plane and through the center	MR^2

PHYSICS

FORMULAE & CONCEPTS

4. Uniform circular ring of radius R
Diameter $MR^2/2$
5. Uniform circular ring of disc of radius R
Perpendicular to its plane and through the center
 $\frac{1}{2}MR^2$
6. Uniform circular ring of disc of radius R
Diameter $\frac{1}{4}MR^2$
7. Hollow cylinder of radius R Axis of cylinder
 MR^2
8. Solid cylinder of radius R Axis of cylinder
 $\frac{1}{2}MR^2$
9. Hollow sphere of radius R
Diameter $\frac{2}{3}MR^2$
10. Solid sphere of radius R Diameter
 $\frac{2}{5}MR^2$

10. Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{m_1 + m_2 + \dots + m_n}$$

$$\vec{a}_{cm} = \frac{1}{M} \cdot \sum_{i=1}^n m_i \cdot \vec{a}_i$$

11. Characteristics of centre of mass

- a. The position of centre of mass depends up on the shape of the body and the distribution of mass
- b. Centre of mass always located towards massive part of the body
- c. Matter may or may not present at centre of mass
- d. If the origin is at the centre of mass sum of moment of masses of the system about centre of mass is zero.
- e. Centre of mass cannot be effected by the internal forces

- f. When no external force acts on a system the velocity of centre of mass of the system remains constant.
- g. The location of the centre of mass is independent of the reference frame used to locate it.

Motion of a cylinder rolling without slipping on an inclined plane

$$a = \frac{mg \sin \theta}{m + I/r^2}$$

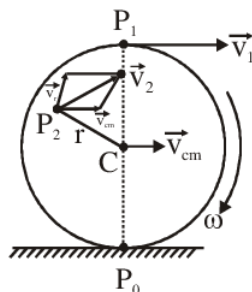
Linear Motion

Rotational Motion

1. Distance/ displacement (s)
2. Angle or angular displacement (θ)
3. Linear velocity, $v = \frac{ds}{dt}$
4. Angular velocity, $\omega = \frac{d\theta}{dt}$
5. Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
6. Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
7. Mass (m)
8. Moment of inertia (I)
9. Linear momentum, $p = mv$
10. Angular momentum, $L = I\omega$
11. Force, $F = ma$
12. Torque = $\tau = I\alpha$
13. Also force, $F = \frac{dp}{dt}$
14. Also, torque = $\tau = \frac{dL}{dt}$
15. Translational K.E. = $\frac{1}{2}mv^2 = \frac{p^2}{2m}$
16. Rotational K.E. = $\frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
17. Work done, $W = Fs$
18. Work done, $W = \tau\theta$
19. Power, $P = Fv$
20. Power, $P = \tau\omega$

12. Rolling Motion

$$\vec{v}_{cm} = R\omega$$

**Kinetic energy of Rolling motion**

$$K = K_T + K_R$$

$$\text{K.E of translation } K_T = \frac{1}{2}mv_{cm}^2$$

$$K_R = \frac{1}{2}I\omega^2$$

$$\text{K.E of rolling body, } K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

$$K = \frac{1}{2}mv_{cm}^2 \left[1 + \frac{k^2}{R^2} \right]$$

13. Centre of gravity

Centre of gravity is a point inside the body through which its whole weight acts

- It refers weight of the body
- At centre of gravity there must be matter present
- Centre of mass and centre of gravity coincide for small bodies and they do not coincide for large bodies like planets

